UMass | Dartmouth
UNIVERSITY STUDIES

Master Syllabus
Course: MTH451
Cluster Requirement: 5A

This University Studies Master Syllabus serves as a guide and standard for all instructors teaching an approved course in the University Studies program. Individual instructors have full academic freedom in teaching their courses, but as a condition of course approval, agree to focus on the outcomes listed below, to cover the identified material, to use these or comparable assignments as part of the course work, and to make available the agreed-upon artifacts for assessment of learning outcomes.

Course Overview: This course will focus on the differential geometry of curves and surfaces.

Learning Outcomes:

Course-Specific Learning Outcomes: Students will know how to describe curves and surfaces in three-dimensions, calculate arc length, curvature and torsion of curves, and will understand the Fundamental Theorem of curves. They will be able to compute and interpret the metric, Christoffel symbols, geodesics and curvatures for standard surfaces, including spheres, saddles, and surfaces of rotation. They will know the distinction between intrinsic and extrinsic properties of curves and surfaces, and the import of Gauss's Theorema Egregium. They will understand the content and have a rough understanding of the proof of the Gauss-Bonnet Theorem. Students will be prepared to study the differential geometry of manifolds.

University Studies Learning Outcomes: In their study of classical topics in differential geometry, students will integrate topics and techniques from vector calculus (curves and surfaces), linear algebra (vectors, quadratic forms, eigenvalues, change of basis), differential equations (existence and uniqueness of solutions for systems of equations), and advanced calculus (continuity, limits, differentiability). They will relate these to the broader University Studies curriculum through their study of the historical development of differential geometry from practical questions in surveying, its ongoing relation to map-making and other aspects of geography, and its applications to physics.

Examples of Texts and/or Assigned Readings: *Elements of Differential Geometry* R. Millman & G. Parker

Example Learning Activities and Assignments:

- Each student will submit written answers, both computations and proofs, to selected textbook problems on a biweekly basis.
- Each student will write a term paper describing a specific application of differential geometry, developing a specific topic related to but not among those covered in the course, or describing the history of the development of some idea within the course. The proposal for the paper topic will include the student's reflection on its role and importance in a larger context. The student will submit and revise a draft of this paper.
- Each student will give an oral presentation based on this paper.

Sample Course Outline:

Week I:
- Introduction
- Review of linear algebra & vector calculus

Week II:
- Regular curves, tangent vectors, reparametrization
- Arc length
- Curvature

Week III:
- Frenet-Serret data
- Frenet-Serret theorem

Week IV:
- Fundamental theorem of curves
- Simple surfaces: open sets, coordinate patches

Week V:
- Tangent planes, unit normals
- Parametric (coordinate) curves

Week VI:
- Surfaces: coordinate patches
- Tangent spaces
- The metric (first fundamental form)

Week VII:
- Normal curvature

Week VIII:
- Geodesic curvature
- Gauss's formulas
- Geodesics

Week IX:
- Parallel transport
- Second fundamental form
- Directional derivatives & the Weingarten map

Week X:
- Principal curvatures
- Gaussian curvature & mean curvature

Week XI:
- Riemannian curvature
- Gauss's equations & the Codazzi-Mainardi equations

Week XII:
- Gauss's *Theorema Egregium*
- Angular variation

Week XIII:
- Gauss-Bonnet formula
- Euler characteristic

Week XIV:
- Gauss-Bonnet theorem